Impact of Input Uncertainty on Failure Prognostic Algorithms: Extending the Remaining Useful Life of Nonlinear Systems

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ABSTRACT

This paper presents a novel set of uncertainty measures to quantify the impact of input uncertainty on nonlinear prognosis systems. A Particle Filtering-based method is also presented that uses this set of uncertainty measures to quantify, in real time, the impact of load, environmental, and other stresses for long-term prediction. Furthermore, this work shows how these measures can be used to implement a novel feedback correction loop aimed to suggest modifications, at a system input level, with the purpose of extending the remaining useful life of a faulty nonlinear, non-Gaussian system. The correction scheme is tested and illustrated using real vibration feature data from a fatigue-driven fault in a critical aircraft component.

1. INTRODUCTION

Particle Filter (PF) algorithms have become a key component of failure prognosis frameworks since they provide a strong mathematical foundation to represent, and even manage, uncertainty in long-term predictions when used in combination with outer feedback correction loops. PF-based prognostic algorithms (Orchard, 2005; Orchard, 2008; Orchard, 2009; Patrick, 2007; Zhang, 2009) have been established as the de facto *state of the art* in failure prognosis, helping to combine the advantages and solving some of the issues that are present in a number of approaches that have been suggested in recent years for uncertainty representation and management in prediction. These methods include probabilistic methods, tools derived from evidential theory or Dempster-Shafer theory (Shafer, 1976), soft-computing methods (fuzzy logic), Confidence Prediction Neural

Networks (NN) (Khiripet, 2001), and probabilistic reliability analysis tools employing an inner-outer loop Bayesian update scheme (Cruse, 2004) to "tune" model hyper-parameters given observations. The implementation of PF-based prognostic frameworks allow to avoid the assumption of Gaussian (or log-normal) probability density functions (PDF) in nonlinear processes, with unknown model parameters, and simultaneously help to consider non-uniform probabilities of failure for particular regions of the state domain. Particularly, the authors in (Orchard, 2008) have proposed a mathematically rigorous method (based on PF, function kernels, and outer correction loops) to represent and manage uncertainty in long-term predictions.

Given that most systems depend on external inputs (and commands), it is particularly important to measure the overall effect that probable future load variations would have on the faulty subsystem under analysis. In order to accurately predict the remaining useful life (RUL) of a system under fault conditions, the prognosis algorithm must take into account the various stresses affecting the system. Stress on the system results from many factors including environmental stresses (wind, temperature, humidity, etc.) and control effort (load, torque, speed etc.). Previous work in failure prognosis has relied on assuming knowledge of these stress conditions. In most applications however, this knowledge is unavailable. Due to the unpredictable nature of environmental and control effort, it benefits the system operator to not only receive prognostics information based on expected stress induced on the system, but information about a range of stress levels, including the most probable stress levels as well as extreme stress levels (i.e. the maximum and minimum stresses). Knowledge of how these varying stress levels affect the remaining useful life of the system provide the operator with a complete picture of how the fault is progressing which will lead to smarter decisions in control to mitigate the fault growth while also meeting the performance require-

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14. ABSTRACT

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ments of the system.

This paper presents a solution for the aforementioned problem and it is organized as follows. Section 2 introduces the basics of particle filtering (PF) and its application to the field of failure prognosis. Section 3 presents the foundations of the analysis of the impact of stress variability on prognostic results and proposes a set of novel uncertainty measures to quantify the aforementioned impact. Section 4 introduces a measure-based feedback correction loop that demonstrates the utility of the measures developed in Section 3 in extending the remaining useful life of a failing system. Section 5 shows the results of the implementation of the proposed feedback correction scheme on real vibration feature data from a fatigue-driven fault test in a critical aircraft component. Finally, Section 6 states the most important conclusions.

2. PARTICLE FILTERING AND FAILURE PROGNOSIS

Prognosis, and thus the generation of long-term predictions, is a problem that goes beyond the scope of filtering applications since it involves future time horizons. Therefore, the application of PF algorithms in a prognosis framework necessarily implies a procedure to project the current particle population in time in the absence of new observations (Orchard, 2009). It is assumed, at this point, that there is at least one feature providing a measure of the severity of the fault condition under analysis (fault dimension); a condition that is necessary for the implementation of any adaptive prognosis scheme. If many features are available, they can always be combined to generate a single signal. In this sense, it is possible to describe the evolution in time of the fault dimension through the nonlinear state equation (Orchard, 2008):

$$\begin{cases} x_1(t+1) = x_1(t) + x_2(t) \cdot F(x(t), t, U) + \omega_1(t), \\ x_2(t+1) = x_2(t) + \omega_2(t) \end{cases} \quad (1)$$

where $x_1(t)$ is a state representing the fault dimension under analysis, $x_2(t)$ is a state associated with an unknown model parameter, U are external inputs to the system (load profile, etc.), $F(x(t), t, U)$ is a general time-varying nonlinear function, ω_1 and ω_2, are white noises (not necessarily Gaussian). For the purposes of this paper the variables $x_1(t)$ and $x_2(t)$ are considered to be scalars, however depending on the implmentation these variable may also represent vectors. The nonlinear function $F(x(t), t, U)$ may represent a model based on first principles, a neural network, or even a fuzzy system.

By using the aforementioned state equation to represent the evolution of the fault dimension in time, it is possible to generate long term predictions using kernel functions to reconstruct the estimate of the state PDF in future time instants:

$$\hat{p}(x_{t+k} | \hat{x}_{1:t+k-1}) \approx$$
$$\sum_{i=1}^{N} w_{t+k-1}^{(i)} K \left(x_{t+k} - E \left[x_{t+k}^{(i)} | \hat{x}_{t+k-1}^{(i)} \right] \right) \quad (2)$$

where $K(\cdot)$ is a kernel density function, which may correspond to the process noise PDF, a Gaussian kernel or

a rescaled version of the Epanechnikov kernel (Orchard, 2008).

The resulting predicted state PDF contains critical information about the evolution of the fault dimension over time. One way to represent that information is through the computation of statistics (expectations, 95% confidence intervals), either the Time-of-Failure (ToF) or the Remaining Useful Life (RUL) of the faulty system. A detailed procedure to obtain the RUL PDF from the predicted path of the state PDF is described and discussed in (Orchard, 2009), although the general concept is as follows. Basically, the RUL PDF can be computed from the function of probability of failure at future time instants. This probability is calculated using both the long-term predictions and empirical knowledge about critical conditions for the system. This empirical knowledge is usually incorporated in the form of thresholds for main fault indicators, also referred to as the hazard zones.

In real applications, it is expected for the hazard zones to be statistically determined on the basis of historical failure data, defining a critical PDF with lower and upper bounds for the fault indicator (H_{lb} and H_{ub}, respectively). Since the hazard zone specifies the probability of failure for a fixed value of the fault indicator, and the weights $\{w_{t+k}^{(i)}\}_{i=1...N}$ represent the predicted probability for the set of predicted paths, then it is possible to compute the probability of failure at any future time instant (namely the RUL PDF) by applying the law of total probabilities, as shown in Eq. (3). Once the RUL PDF is computed, combining the weights of predicted trajectories with the hazard zone specifications, it is well known how to obtain prognosis confidence intervals, as well as the RUL expectation.

$$\hat{p}_{TTF}(t) =$$
$$\sum_{i=1}^{N} \Pr(Failure | X = \hat{x}_t^{(i)}, H_{lb}, H_{ub}) \cdot w_t^{(i)} \quad (3)$$

Equations (1), (2), and (3) can be used to show that the *a priori* state PDF for future time instants, and thus the time-of-failure (ToF) PDF, directly depends on the *a priori* probability distribution of the load profile for future time instants. Most of the times, long-term predictions assume that the latter distribution is a Dirac's delta function, which basically implies a deterministic function of time for future load profiles. Although this simplification helps to speed up the prognostic procedure and to generate the most likely ToF estimate, it does not consider future changes in operating conditions or unexpected events that could affect the remaining useful life of the system under analysis. Monte Carlo simulation can be used to generate ToF estimates for arbitrary *a priori* distributions of future load conditions, however it is not always possible to obtain these results in real-time. In this sense, PF-based prognostic routines not only provide a theoretical framework where these concepts can be incorporated in real-time, but also allow the use of uncertainty measures to characterize the sensitivity of the system with respect to changes in future load distributions.

Furthermore, if a formal definition of mass probability is assigned to each possible stress condition, a ToF PDF estimate can be obtained as a weighted sum of kernels, where each kernel represents the PDF estimate of a

known constant load. Indeed, if the *a priori* distribution of future operating conditions is given by:

$$P\{U = u\} = \sum_{j=1}^{N_u} \pi_j \delta(u - u_j), \qquad (4)$$

where $\{u_j\}_{j=1}^{N_u}$ is a set of constant load values, then the probability of failure at a future time t can be computed using (5).

$$\hat{p}_{ToF}(t) = \sum_{j=1}^{N_u} \pi_j \sum_{i=1}^{N} w_t^{(i)} \cdot P\left(\hat{x}_t^{(i)}\right), \qquad (5)$$

$$P\left(\hat{x}_t^{(i)}\right) = \Pr(Failure | X = \hat{x}_t^{(i)}, U = u_j, H_{lb}, H_{ub})$$

3. IMPACT OF VARYING STRESS LEVELS ON A FAULTY SYSTEM

Figure 3. illustrates the predicted fault growth of a system where a fault is detected at time t_{detect} and a prediction of the remaining useful life is made at time $t_{prognosis}$ for varying stress levels. The illustration shows how varying the stress on the system can have a significant impact on its remaining useful life. The objective is to characterize the manner in which the stress uncertainty affects the uncertainty in the RUL estimates.
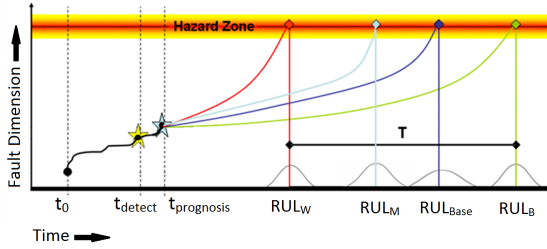

Figure 1: Predicted Fault Growth Curves for Maximum, Minimum, Median, and Baseline Stress Levels.

Consider, for instance, the growth of a given fault condition when the system is affected by four different stress profiles, see Figure 1: the expected or baseline stress level (U_{Base}), the maximum or worst-case stress level (U_W), the minimum or best-case stress level (U_B), and the midpoint between the best and worst-case stress levels (U_M). The resulting RUL prediction for each of these stress levels are RUL_{Base}, RUL_W, RUL_B, and RUL_M respectively, as defined below.

RUL$_{Base}$	**:**	PDF of the predicted RUL for the baseline load (most likely stress level)
RUL$_B$	**:**	PDF of the predicted RUL for the minimum load (best-case stress level)
RUL$_W$	**:**	PDF of the predicted RUL for the maximum load/overload (worst-case stress level)
RUL$_M$	**:**	PDF of the predicted RUL for the median load (mean of best and worst case)

It is assumed that the trivial stress profile (constant null load) case is infeasible and a set of feasible stress profiles and operating points is given. The assumption to ignore the trivial stress profile can be made because such a stress profile indicates a system that is not in operation (e.g. an aircraft cannot stay aloft without a non-zero level of stress being exerted on the system).

The analysis of the best and worst case stress profiles creates a range of RUL predictions that are used to provide bounds on the prediction horizon. From this range, predictions based on the expected or baseline stress conditions can be compared in order provide a context for understanding how future changes in the stress on the system will influence its remaining useful life. Measures that encapsulate the effects of varying the stress on the system are created and discussed below.

3.1 Load Prediction Index

The Load Prediction Index (*LPI*), is a measure of how close the baseline RUL prediction is to the extreme RUL cases (RUL_W and RUL_B). The Load Prediction Index is defined in Equation (7). The utility of the Load Prediction Index is that it converts the unbounded number that is the mean value of the predicted RUL and normalizes it to a value, $LPI \in [0, 1]$, where an *LPI* near 0 indicates that the baseline RUL prediction is very near the best case stress level and little improvement in RUL can be obtained through decreasing the stress level. Conversely an *LPI* near 1 indicates an RUL near the worst case stress level and improvement is possible through decreasing the stress level. This normalization allows for the creation of generic algorithms to handle control requests based on anticipated outcomes of adjusting the stress.

$$T = Mean(RUL_B) - Mean(RUL_W) \qquad (6)$$

$$LPI = \frac{Mean(RUL_B) - Mean(RUL_{Base})}{T} \qquad (7)$$

3.2 Load Prediction Percentage

The Load Prediction Percentage (*LPP*) is an indicator of precision in our predicted RUL PDF with respect to the best and worst case predictions. The *LPP* is the percentage of T that is covered by the standard deviation of the RUL estimate. The utility of the *LPP* is that it converts the standard deviation of a predicted RUL, a number that varies significantly for different applications, and converts it to a number that is a function of RUL_B and RUL_W. This normalizes the standard deviation so that it can be compared with other RUL standard deviations from various applications and prognosis algorithms.

$$LPP = \frac{stdev(RUL_{Base})}{T} \qquad (8)$$

3.3 Maximum Dispersion

The Maximum Dispersion (MD) metric is a measure of how much uncertainty in the stress profile (U) the system can tolerate, while still ensuring operation bounded by

either the worst or best case scenarios. MD is defined as follows:

$$MD = min(\upsilon) \text{ s.t.}$$
$$Pr\{rul \in RUL \cap RUL_B\} \geq c \text{ or}$$
$$Pr\{rul \in RUL \cap RUL_W\} \geq c \quad (9)$$

Where υ is the variance of the stress input that is associated with the RUL PDF estimate and $c \in [0, 1]$.

As the uncertainty of the load increases, the uncertainty of the resulting RUL PDF estimate will also increase. MD shows a bound for the load uncertainty that limits the overlap between the resulting RUL estimate and either the best or worst case scenarios. The utility of this metric is that it provides a boundary for the level of noise that can be tolerated in the prediction. A level of noise that is beyond the maximum dispersion will lead to an unreliable prediction.
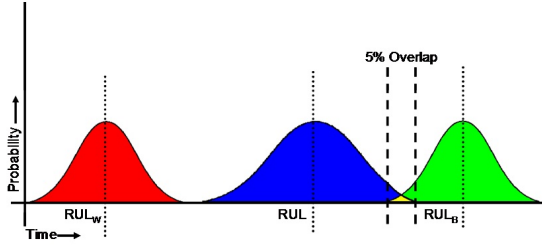

Figure 2: PDF of RUL overlapping with best case RUL PDF.

3.4 Stress Sensitivity Measures

Stress sensitivity is a measure of the change in uncertainty in the RUL prediction as a function of the uncertainty in the stress profile. Stress sensitivity is found by adding a Gaussian white noise to the median stress level and comparing the resulting RUL PDF with the RUL PDF from the deterministic median stress level. This effect is illustrated in Figure 3 where the green kernels show U_{Base} and the resulting RUL_{Base} PDF, and the blue kernels show $U_{Base+\omega}$ and the resulting $RUL_{Base+\omega}$ PDF. Since U_{Base} is a deterministic function of time, then its *a priori* distribution is represented as a Dirac's delta function. Stress sensitivity is measured in two ways, Dispersion Sensitivity, defined in Eq. (10) and Confidence Interval Sensitivity, defined in Eq. (11)
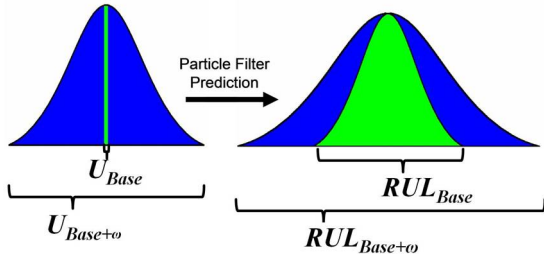

Figure 3: Stress Sensitivity.

Dispersion Sensitivity:

$$DS_\omega = \frac{stdev(RUL_{Base+\omega})}{stdev(RUL_{Base})} \quad (10)$$

Confidence Interval Sensitivity:

$$CIS_\omega = \frac{Length(CI\{RUL_{Base+\omega}\})}{Length(CI\{RUL_{Base}\})} \quad (11)$$

where $RUL_{M+\omega}$ is the predicted RUL with a load factor of $U_{M+\omega}$ where $U_{M+\omega}(t) = U_M(t) + \omega(t)$ and $\omega(t)$ is Gaussian white noise. The stress sensitivity measures provide a means of determining how adjustments in stress will affect the RUL prediction without the need of running individual simulations for each potential stress. Low stress sensitivity measures indicate a system which has a remaining useful life that will vary little with respect to changes in the load profile whereas a high stress sensitivity measure indicates an ability on the part of the operator or reconfigurable controller alter the RUL through adjustments in the load profile.

4. UNCERTAINTY MEASURE-BASED FEEDBACK CORRECTION LOOPS FOR EXTENSION OF REMAINING USEFUL LIFE

The main motivation behind the definition of uncertainty measures, based on the outcomes from PF-based prognostic routines, is to characterize the effects that changes in operating conditions may have on the resulting remaining useful life of the system, in real-time. However, that is only the first step in a more complex problem: to establish correction loops aimed to extend the remaining useful life of a piece of equipment. This section of the paper presents and analyzes a novel measure-based method that is proposed as a general approach to establish feedback correction loops aimed to lengthen the RUL of a nonlinear system. The method utilizes a PF-based prognosis framework to determine the baseline PDF estimate of the remaining useful life (RUL_{Base}) and then utilizes the sensitivity measures (DS and CIS) to determine an appropriate stress level that will extend the RUL of the component to the specified desired RUL (RUL_d). Two approaches to the methodology are outlined below, the DS-based Approach and the CIS-based Approach. It should be noted that this feedback correction loop is discussed to demonstrate that potential of the uncertainty measures. A rigorous study of the stability of this feedback correction routine must be undertaken before it can be implemented in a real-world scenario.

DS-based Approach to RUL Extension

Given a baseline RUL (RUL_{Base}), determined through PF-based prognostic routines from a baseline stress level of U_{Base}, knowledge of the Dispersion Sensitivity allows the system operator to extend the RUL from RUL_{Base} to RUL_d by adjusting the stress factor to a safe level (U_d). To determine U_d, the standard deviation of the RUL prediction which places RUL_d in the 95th percentile of the distribution, while maintaining a mean of $mean\{RUL_{Base}\}$, must be determined. This distribution is denoted as $RUL_{Base+\bar{\omega}}$, as shown in (12). Using a linear fit to map the standard deviation of the stress to the standard deviation of the remaining useful life, the standard deviation of the stress profile required to output

a distribution of $RUL_{Base+\bar{\omega}}$ is determined by (13). The standard deviation of this stress is then utilized to determine how much the baseline stress must be reduced in order to attain a remaining useful life of RUL_d, as seen in (14).
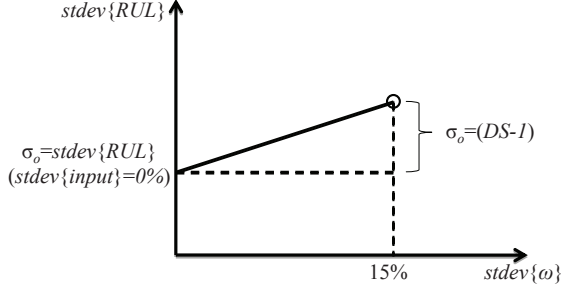

Figure 4: Linear Mapping Between Standard Deviation of Stress and Standard Deviation of RUL.

$$stdev\{RUL_{Base+\bar{\omega}}\} =$$
$$\frac{RUL_D - E\{RUL_{Base}\}}{Z_{0.95}} \quad (12)$$

$$stdev\{U_{Base+\bar{\omega}}\} =$$
$$\left(\frac{stdev\{RUL_{Base+\bar{\omega}}\}}{stdev\{RUL_{Base}\}} - 1 \right) \frac{stdev\{\omega\}}{DS - 1} \quad (13)$$

$$U_d = U_{Base} - stdev\{U_{Base+\bar{\omega}}\} \quad (14)$$

CIS-based Approach to RUL Extension

Similar to the dispersion sensitivity approach, given RUL_{Base} from a baseline stress level of U_{Base}, knowledge of Confidence Interval Sensitivity allows the system operator to extend the RUL from RUL_{Base} to RUL_d by adjusting the stress factor to a safe level (U_d). To determine U_d, the confidence interval length of the RUL prediction which places RUL_d at the highest end of the confidence interval of the distribution, while maintaining a mean of $mean\{RUL_{Base}\}$, must be determined. This distribution is denoted as $RUL_{Base+\bar{\omega}}$, as shown in (15). Using a linear fit to map the confidence interval length of the stress to the confidence interval length of the remaining useful life, the standard deviation of the stress required to output a distribution of $RUL_{Base+\bar{\omega}}$ is determined by (16). The standard deviation of this stress is then utilized to determine how much the baseline stress must be reduced in order to attain a remaining useful life of RUL_d, as seen in (17).

$$length\left(CI\{RUL_{Base+\bar{\omega}}\}\right) =$$
$$2\left(RUL_D - E\{RUL_{Base}\}\right) \quad (15)$$

$$stdev\{U_{Base+\bar{\omega}}\} =$$
$$\left(\frac{length(CI\{RUL_{Base+\bar{\omega}}\})}{length(CI\{RUL_{Base}\})} - 1 \right) \frac{stdev\{\omega\}}{CIS - 1} \quad (16)$$

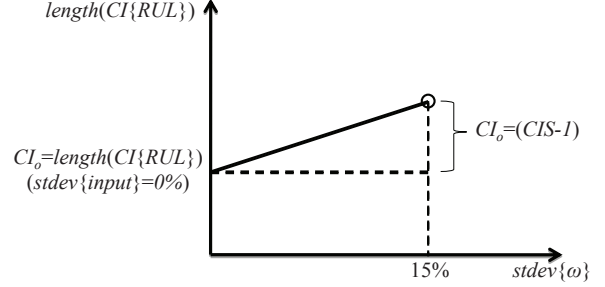$$U_d = U_{Base} - stdev\{U_{Base+\bar{\omega}}\} \quad (17)$$

Figure 5: Linear Mapping Between Length of the Confidence Interval of Stress and Length of the Confidence Interval of RUL.

5. CASE STUDY: LOAD REDUCTIONS AND ITS EFFECT ON FATIGUE CRACK GROWTH

An appropriate case study has been designed to test and show the potential of the proposed feedback correction strategy. This case study uses data (from a seeded fault test) that describes a propagating fatigue crack on a critical component in a rotorcraft transmission system, which emulates a situation where the pilot must remain airborne for a given amount of time in order to reach a safe landing destination. In this example the remaining useful life is defined as the time remaining until the failing compont is no longer operable and the system encounters catastrophic failure. The RUL extension methods discussed in this section will provide the pilot, or reconfigurable controller, with the information needed to adjust the load of the aircraft and reduce the stress on the failing component, with the purpose of extending the RUL to a desired time that ensures safe landing. Although a physics-based model for a system of these characteristics is a complex matter, it is possible to represent the growth of the crack (fault dimension) using a simplified model, where some nonlinear mapping functions are defined on the basis of an ANSYS stress model for the inner and outer tips of the fatigue crack (Orchard, 2009 and Patrick, 2007).

In the experiment, the baseline stress level was 120% of the maximum recommended torque. If this information is fed into the proposed PF-based prognosis framework, then the resulting ToF PDF (see cyan PDF in Figure 6), computed at the 300th cycle of operation, has an expectation of 594 cycles, a standard deviation of 12.44 cycles, and a confidence interval length of 38 cycles for $\alpha = 95\%$. If we are to compute the DS and CIS measures for this system at that particular cycle of operation (300th cycle), then it is necessary to compute the statistics of the ToF PDF that results after including uncertainty in the system input. Given that the implementation of a PF-based framework for failure prognosis allows to perform this task in a simple and efficient manner, it is possible, for example, to analyze the case when the input uncertainty is characterized by zero-mean Gaussian noise (standard deviation of 15% of maximum recommended torque). The resulting ToF PDF, has a standard deviation of 41.52 cycles and a confidence interval length of 142 cycles for $\alpha = 95\%$ (see magenta PDF in Figure 6). Considered the aforementioned information,

the dispersion sensitivity is found to be:

$$DS_{15\%} = \frac{stdev\{RUL_{Base+\omega}\}}{stdev\{RUL_{Base}\}} =$$

$$\frac{41.52 cycles}{12.44 cycles} = 3.3362$$

and the confidence interval sensitivity is computed as

$$CIS_{15\%} = \frac{length(CI\{RUL_{Base+\omega}\})}{length(CI\{RUL_{Base}\})} =$$

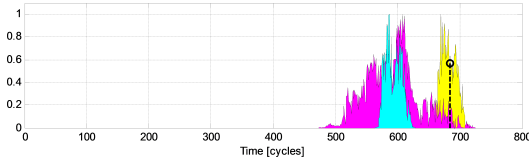$$\frac{142 cycles}{38 cycles} = 3.7368.$$

Figure 6: ToF Distributions for Baseline, Noisy, and Desired Stress Levels for a Cracked Gear Plate.

For this system the desired ToF is 714 cycles (RUL of 414 cycles). If we were to use the DS-based approach to RUL extension to suggest a correction in the stress profile for the system, then the standard deviation of the noise level required for cycle 714 to be located at the 95th percentile of the predicted magenta ToF PDF is found by:

$$stdev\{RUL_{Base+\bar{\omega}}\} =$$

$$\frac{RUL_D - E\{RUL_{Base}\}}{Z_{0.95}} =$$

$$\frac{714 - 594}{1.627} = 73.755$$

Inserting this value into (13) and solving for $stdev\{U_{Base+\bar{\omega}}\}$ yields a required standard deviation of 31.64% for the input stresses. Therefore in order to achieve the desired RUL of 714 cycles, the stress factor must be reduced by 31.64% from 120% to 88.36%. Similarly, for the CIS-based approach to RUL extension, it is possible to estimate the required variation considering:

$$length(CI\{RUL_{Base+\bar{\omega}}\}) =$$

$$2\left(RUL_D - E\{RUL_{Base}\}\right) =$$

$$2(714 - 594) = 240$$

Inserting this value into (16) and solving for $stdev\{U_{Base+\bar{\omega}}\}$ yields a required standard deviation of 29.13% for the input stress. Therefore in order to achieve the desired RUL of 714 cycles, the stress factor must be reduced by 29.13% from 120% to 90.70%. Compare 88.36% and 90.70% to the actual stress factor that results in a RUL of 714, which is 93%. Clearly, both approaches for stress correction suggest a modification, for the system input, that would have translated in an appropriate extension of the remaining useful life of the system.

6. CONCLUSION

This paper presents and tests a general approach for a novel feedback correction loop, based on uncertainty measures, to lengthen the RUL of a nonlinear, non-Gaussian system. The method searches for a linear relationship between the amount of uncertainty in the input of a nonlinear stochastic system and the one that can be found on its RUL estimate. Although the feedback loop is implemented using simple linear relationships, it is helpful to provide a quick insight into the manner that the system reacts to changes on its input signals in terms of its predicted RUL. The method is able to manage non-Gaussian PDF's since it includes concepts such as nonlinear state estimation and confidence intervals in its formulation. Real data from a fault seeded test was used to check if the proposed framework was able to anticipate modifications on the system input to lengthen its RUL. Results of this test indicate that the method was able to successfully suggest the correction that the system required. Future work will be focused on the development and testing of similar strategies using different input-output uncertainty metrics.

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